"Mathematics of Sound: The Architecture of Acoustics"

Back Row (left to Right): Xavier Maciel, Matthew Sarrico, Carlos Viera, Freddy Pastrano, Bruno Coimbra, Mr. Henry Varum
Front Row (Left to Right): Nichole Bermudez, Hallana Trinetta, Maria Silva, Daniella Dias, Kelly Amorin

Certificate of Participation

This certificate is awarded to

Ann Street School
Grade 8 - Mr. Varum

For creating a project for the 2009 Mathematics Expo

Newark Public Schools
Office of Mathematics

May Samuels
Signature

5/29/09
Date

2350-04—Hammond & Stephens, Fremont, NE

"Mathematics of Sound: "The Architecture of Acoustics"

Room 407 was filled with curious minds, as we Algebra students pondered about the topic for the math fair. We grew anxious yet nervous, because this topic was both fascinating and new to us. Mr. Varum had constructed exemplary projects in the previous years, and how could we reach those standards with a topic so new to our young minds?

Much research was done about the frequencies of notes, and the direct relationships that it has with string lengths. Pythagoras had discovered the relationship between these frequencies, and how they related to string lengths. This idea that Pythagoras had presented to us gave us a strong understanding of what we wanted the basis of our project to be about. Eyes lit up as our minds had all interconnected. The idea of building a guitar had hit us in the head. We knew that this idea would challenge the minds of us individuals, but this was a challenge that we were willing to meet. Since a harp also had a direct relationship from string length to note frequencies, we had decided that developing a model for a harp would be an addition to the foundation that we had already built.

With an idea set, we had gotten our pencils and lifted our graph paper and got down to work. Our work consisted of creating a design for both the harp and the guitar. During that process, we used scale models and figured out the sectors and the areas of various shapes. In the making of these models we also used tangent, sine, and cosine ratios to precisely shape this guitar. We mathematically found arc lengths that would help in the outline of the guitar. Modeling and scaling the guitar and the harp was only one key component in the multiple steps in building a guitar. Exponential growth and decay was used to establish note frequencies, and string lengths. There was a certain ratio that would be used in both situations, that interconnects the notes with the strings. Proportional reasoning and combinations of metric and standard system of measurements were used in building the guitar. Much effort and teamwork was put into the making of this project. We have a more clear understanding of the mathematics behind music, and we are sure that we will bring this knowledge with us, and share with others our appreciation for fine music.

Harp Design and Mathematics

Lengths of the Harps Strings

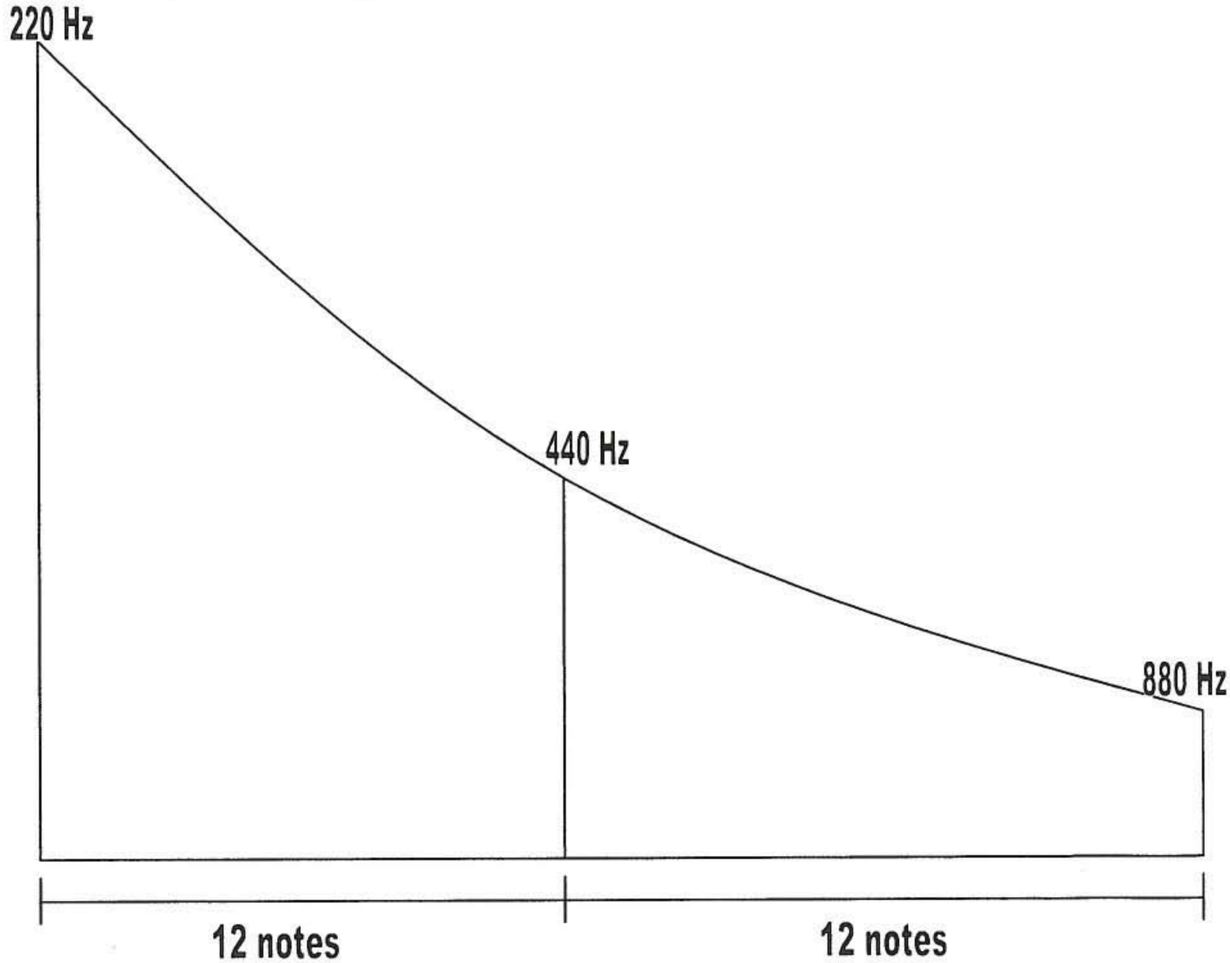
We came to a realization that the frequencies had a relationship from one note to another. The note below was based on the note above to establish its frequency. When the note below was divided with the note above, it gave the common ratio between the frequencies. We had concluded that this was the correct ratio after we had experimented on many different notes from ***C3-C6.*** This ratio is ***.9438743127.*** After we had encountered this ratio, we realized that the string lengths or our harp must have the same ratio as the frequencies had, because the frequency and length ratios are all interconnected. We wanted to base the lowest note on a string length of 1,000 mm, for this was a solid number that is easy to work with. To discover the string lengths of the following notes, we multiplied the 1,000 mm, by .9438743127. We continued this same process until we had reached note C6, which is the highest note, but the shortest string. This was our method that we developed to find the string lengths.

$\underline{1000(.9438743127)^X}$

Formula used for this process.

Formulas for Exponential Growth and Decay of String Lengths and Frequencies

According to Pythagoras if you have a string that is of a specific tension and halve it, the frequency of the smaller string will be double the frequency of the original.



If you attempt to connect these points you realize that this is an example of exponential growth and decay.

Since there are 12 notes in our scale a formula can be used to find the frequencies of these notes:

$$220(x)^{12} = 440$$

$$220x^{12} = 440$$

$$X^{12} = 2$$

$$\sqrt{x^{12}} = \sqrt{2}$$

$$X = 1.059463094$$

FORMULA TO FIND FREQUENCY GROWTH:

$$220(1.059463094)^{X}$$

X= note number after 220 Hz (C3)

A formula can be used to find the string lengths of the harp.

$$440(x)^{12} = 220$$

$$440x^{12} = 220$$

$$X^{12} = .5$$

$$X = .9438743127$$

FORMULA TO FIND STRING LENGTH

$$\mathbf{1000(.94238743127)^{x}}$$

X= note after first string (C3)

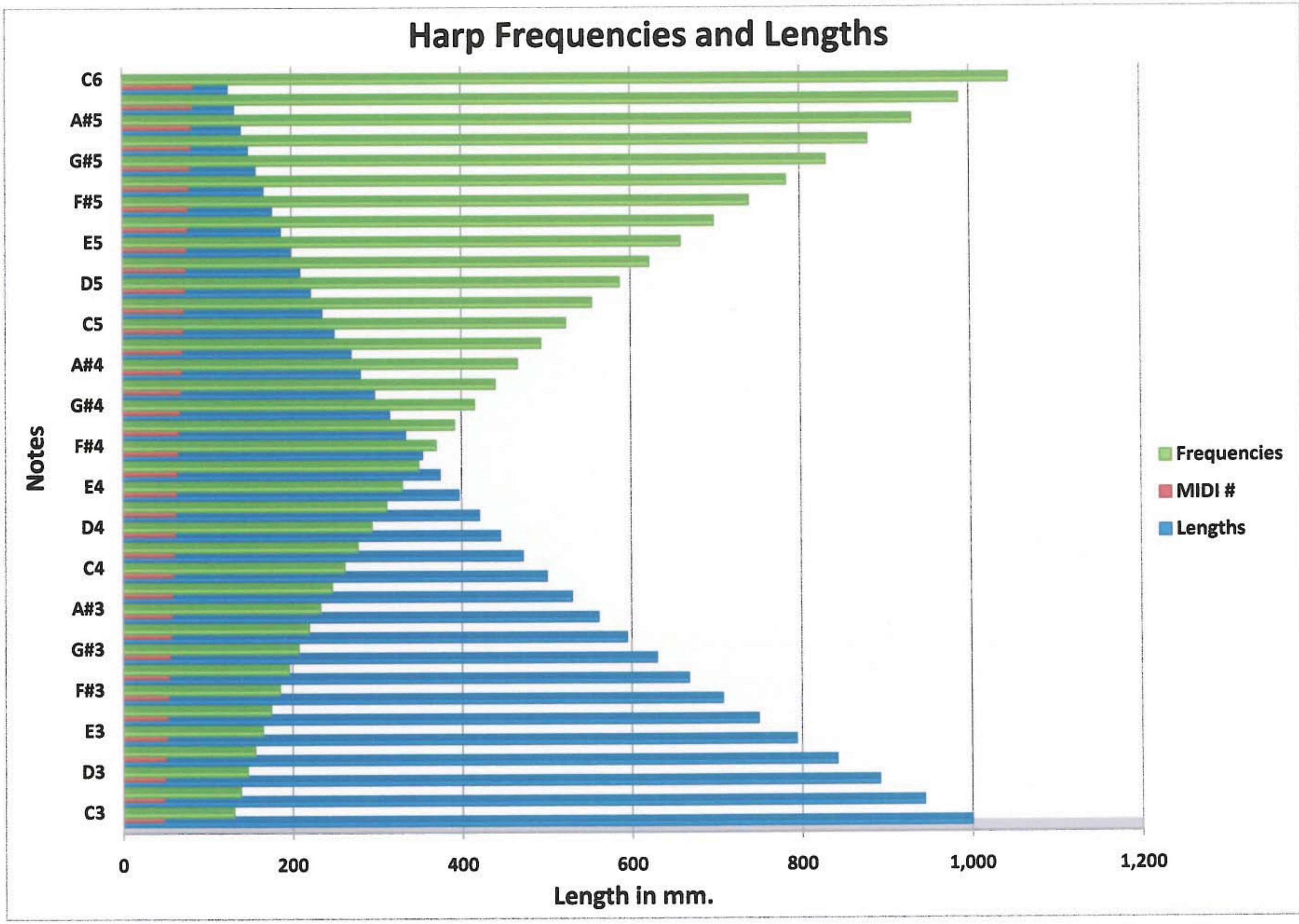
THE MATHEMATICS BEHIND THE LENGTHS AND FREQUENCIES OF A HARP

Notes	MIDI Number	Frequencies	Lengths
C3	48	6.875*2^((3+84)/12)= 130.81278265	1,000 mm
C#3	49	6.875*2^((3+83)/12)= 138.591315488	1,000*.9438743127=943.8743127 mm
D3	50	6.875*2^((3+82)/12)= 146.832383959	943.8743127*.9438743127=890.8987182 mm
D#3	51	6.875*2^((3+81)/12)= 155.563491861	890.8987182*.9438743127=840.8964153 mm
E3	52	6.875*2^((3+80)/12)= 164.813778456	840.8964153*.9438743127=793.700526 mm
F3	53	6.875*2^((3+79)/12)= 174.614115717	793.700526*.9438743127=749.1535385 mm
F#3	54	6.875*2^((3+78)/12)= 184.997211356	749.1535385*.9438743127=707.1067813 mm
G3	55	6.875*2^((3+77)/12)= 195.997717991	707.1067813*.9438743127=667.4199272 mm
G#3	56	6.875*2^((3+76)/12)= 207.65234879	667.4199272*.9438743127=629.960525 mm
A3	57	6.875*2^((3+75)/12)= 220	629.960525*.9438743127=594.6035576 mm
A#3	58	6.875*2^((3+74)/12)= 233.081880759	594.6035576*.9438743127=561.2310293 mm
B3	59	6.875*2^((3+73)/12)= 246.9416506	561.2310293*.9438743127=529.7315473 mm
C4	60	6.875*2^((3+72)/12)= 261.625565301	529.7315473*.9438743127=500 mm
C#4	61	6.875*2^((3+71)/12)= 277.182630977	500*.9438743127=471.9371565 mm
D4	62	6.875*2^((3+70)/12)= 293.664767917	471.9371565*.9438743127=445.4493592 mm
D#4	63	6.875*2^((3+69)/12)=311.126983722	445.4493592*.9438743127=420.4482077 mm
E4	64	6.875*2^((3+68)/12)= 329.627556913	420.4482077*.9438743127=396.8502631 mm
F4	65	6.875*2^((3+67)/12)= 349.228231433	396.8502631*.9438743127=374.567693 mm
F#4	66	6.875*2^((3+66)/12)= 369.994422712	374.567693*.9438743127=353.5533907 mm
G4	67	6.875*2^((3+65)/12)= 391.995435982	353.5533907*.9438743127=333.7099637 mm
G#4	68	6.875*2^((3+64)/12)= 415.30469758	333.7099637*.9438743127=314.9802626 mm
A4	69	6.875*2^((3+63)/12)= 440	314.9802626*.9438743127=297.3017789 mm
A#4	70	6.875*2^((3+62)/12)= 466.163761518	297.3017789*.9438743127=280.6155122 mm
B4	71	6.875*2^((3+61)/12)= 493.883301256	280.6155122*.9438743127=264.8657737 mm
C5	72	6.875*2^((3+60)/12)= 523.251130601	264.8657737*.9438743127=250 mm
C#5	73	6.875*2^((3+59)/12)= 554.365261954	250*.9438743127=235.9685783 mm

C#5	73	6.875*2^((3+59)/12)= 554.365261954	250*.9438743127=235.9685783 mm
D5	74	6.875*2^((3+58)/12)= 587.329535835	235.9685783*.9438743127=222.7246796 mm
D#5	75	6.875*2^((3+57)/12)= 622.253967444	222.7246796*.9438743127=210.2241039 mm
E5	76	6.875*2^((3+56)/12)= 659.255113826	210.2241039*.9438743127=198.4251316 mm
F5	77	6.875*2^((3+55)/12)= 698.456462866	198.4251316*.9438743127=187.2883847 mm
F#5	78	6.875*2^((3+54)/12)= 739.988845423	187.2883847*.9438743127=176.7766954 mm
G5	79	6.875*2^((3+53)/12)= 783.990871963	176.7766954*.9438743127=166.8549819 mm
G#5	80	6.875*2^((3+52)/12)= 830.60939516	166.8549819*.9438743127=157.4901313 mm
A5	81	6.875*2^((3+51)/12)= 880	157.4901313*.9438743127=148.6508895 mm
A#5	82	6.875*2^((3+50)/12)= 932.327523036	148.6508895*.9438743127=140.3077561 mm
B5	83	6.875*2^((3+49)/12)= 987.766602512	140.3077561*.9438743127=132.4328869 mm
C6	84	6.875*2^((3+48)/12)= 1046.5022612	132.4328869*.9438743127=125 mm

String Length Formula= 1000(.9438743127)^X

The formula that was used to find the string lengths is 1000(.9438743127)^X. This equation might seem quite complex, but it is in fact just a matter of interpretation. The 1,000 represents the string length of the first string (note C3) in millimeters. The .9438743127 represents the ratio between each note, and each string length. The X represents the string that you want to find the length of. If you were to multiply the equation to the power of 1, you would encounter your second string length, and the power of 2 would result in the string length of the 3rd string, etc. We had used this equation until the power of 36, which would give you the last string (37th string, note C6).


Harp Frequencies and Lengths
C6
A#5
G#5
F#5
E5
D5
C5
A#4
G#4
F#4
E4
D4
C4
A#3
G#3
F#3
E3
D3
C3
Notes
0
200
400
600
800
1,000
1,200
Length in mm.
Frequencies
MIDI #
Lengths

Math Behind Our Harp Design

We wanted our harp design to have the appearance of most other harps, so instead of designing a harp with a flat sound box, parallel to the floor, we decided to angle our sound box, like most harps. We were pleasantly surprised to see that the curve created by the 50.22702036° angle of the sound box was similar in shape to other harps.

Mathematics Behind the Harp Design

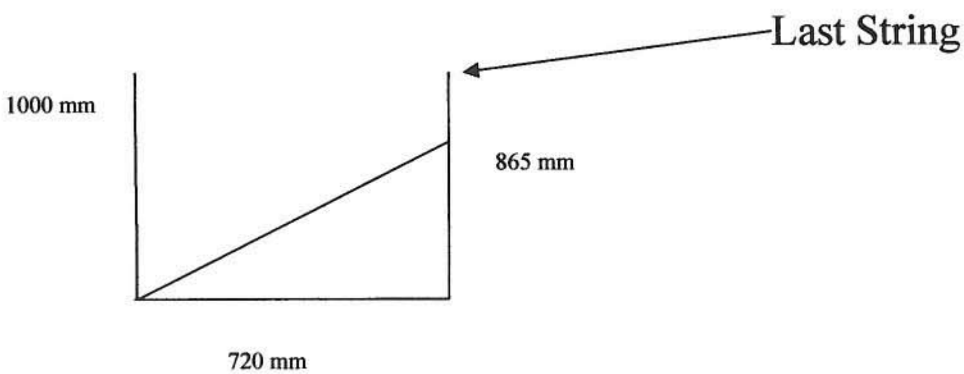
Finding the Angle of Sound Box

-The first string is 1000 mm.

-The last string is 125 mm.

-We want the height of the 1^{st} and last string to differ by 10 mm.

-This means that if the last string is 990 mm high the base is 865 mm off the floor.



Each string is 2 cm apart on center.

370x2=740   740-20=720 mm ← Distance of the last string from the 1^{st} string.

* $\tan^{-1}(865/720) = 50.22702036^{\circ}$ = **angle of the sound box**

MAXIMUM AND MINIMUM STRING HEIGHTS

String	MAXIMUM HEIGHT	MINIMUM HEIGHT
C3	1024.02777778 mm	24.02777778 mm
C#3	991.92986826 mm	48.05555556 mm
D3	962.982051515 mm	72.08333334 mm
D#3	937.007526423 mm	96.11111112 mm
E3	913.839414946 mm	120.1388889 mm
F3	893.320205191 mm	144.166666668 mm
F#3	875.301225729 mm	168.19444446 mm
G3	859.642149416 mm	192.22222224 mm
G#3	846.210525065 mm	216.25000002 mm
A3	834.881335405 mm	240.277778 mm
A#3	825.536579844 mm	264.30555558 mm
B3	818.064880653 mm	288.33333336 mm
C4	812.361111256 mm	312. 36111114 mm
C#4	808.32604538 mm	336.38888892 mm
D4	805.866025891 mm	360.4166667 mm
D#4	804.892652229 mm	384.44444448 mm
E4	805.322485375 mm	408.47222226 mm
F4	807.076769383 mm	432.50000004 mm
F#4	810.081168537mm	456.52777782 mm
G4	814.265519265mm	480.5555556 mm
G#4	819.563595976 mm	504.58333338 mm
A4	825.912890032 mm	528.61111116 mm
A#4	833.254401137 mm	552.63888894 mm
B4	841.532440428 mm	576.66666672 mm
C5	850.694444616 mm	600.694445 mm
C#5	860.690800565 mm	624.72222228 mm
D5	871.472679707 mm	648.75000006 mm
D#5	883 mm	672.77777784 mm
E5	895.230687224 mm	696.80555562 mm
F5	908.121718115 mm	720.8333334 mm
F#5	921.63780658 mm	744.111118 mm
G5	935.743870832 mm	769.88888896 mm
G#5	950.406798075 mm	792.91666674 mm
A5	965.59533399 mm	816.94444452 mm
A#5	981.279978431 mm	840.9722223 mm
B5	997.432886965 mm	865
C6	1014	889.03 mm

String Height

Tan(50.22702036)= x/2

1.201388889=x/2

2.402777778=x

For every 2 cm, the harp's sound box rises by 2.40277777 cm or 24.02777778 mm.

Height of C3= 1000+24.02777778=1024.02777778

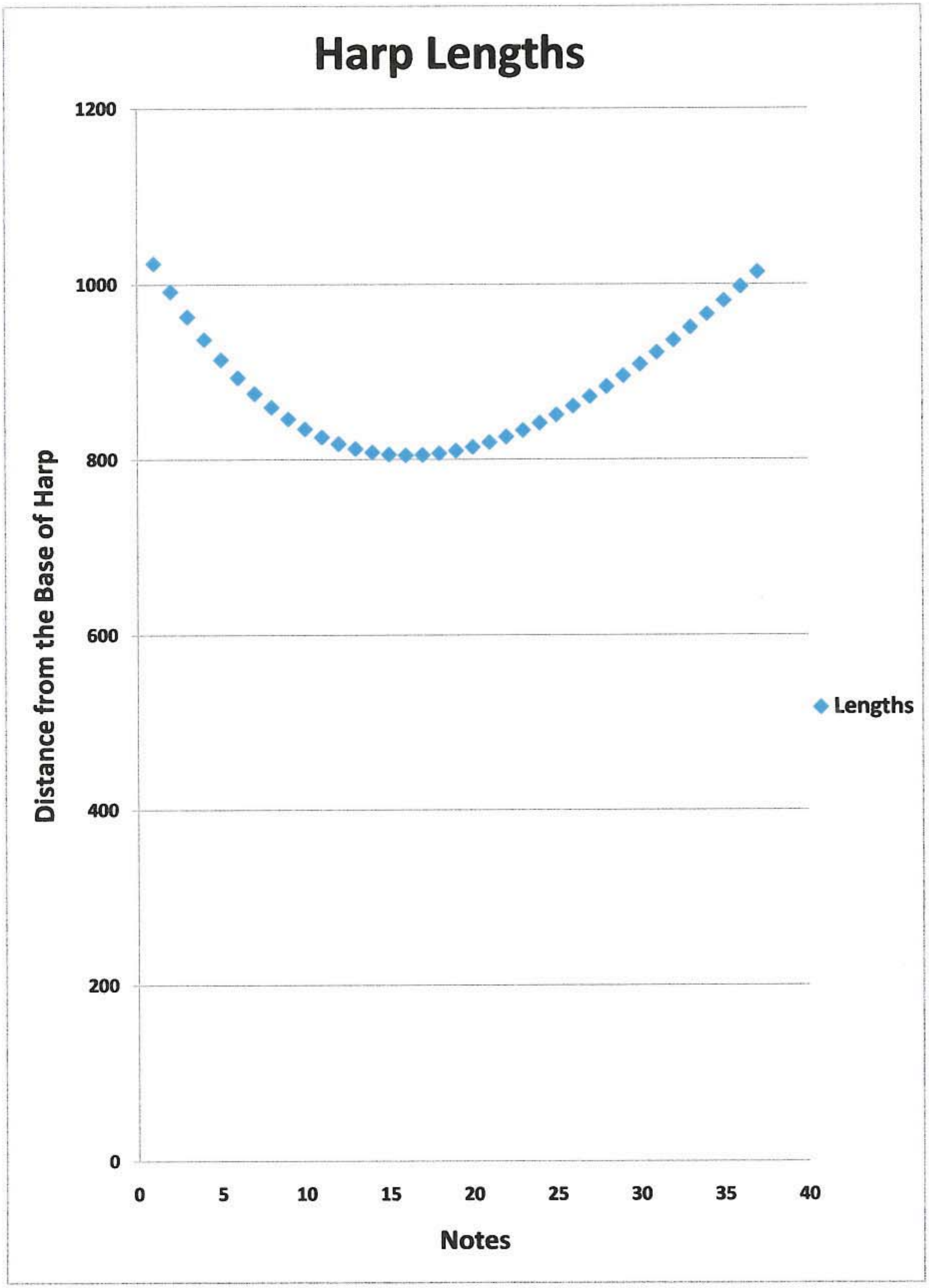
Height of C4= 943.8743127+24.02777778=991.9298683

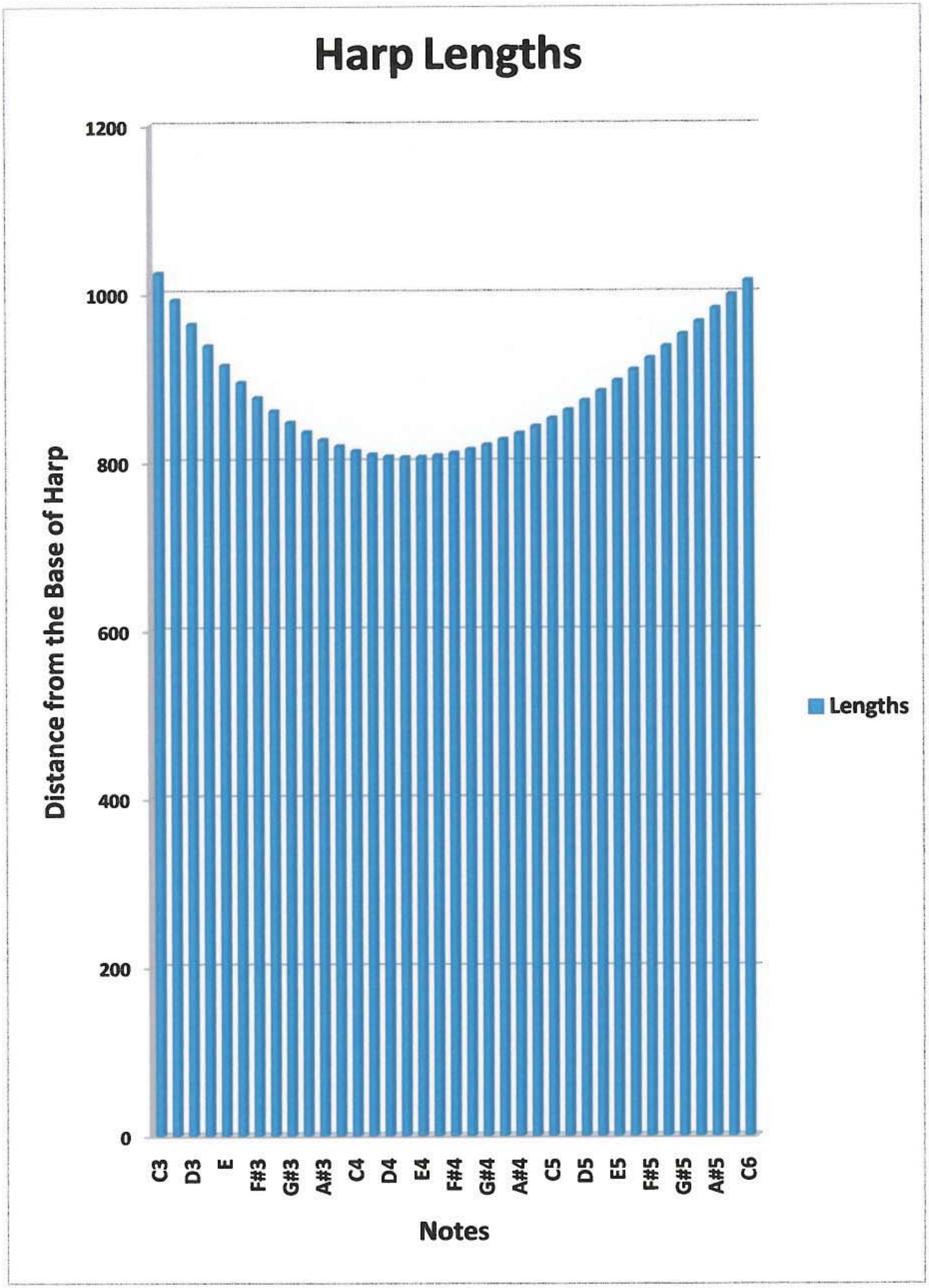
This pattern shows us that a formula can be used to find the string lengths.

String length + distance off of the ground= max height of each string

$(1000(.9438743127)^x)+(24.0277777(x+1))$

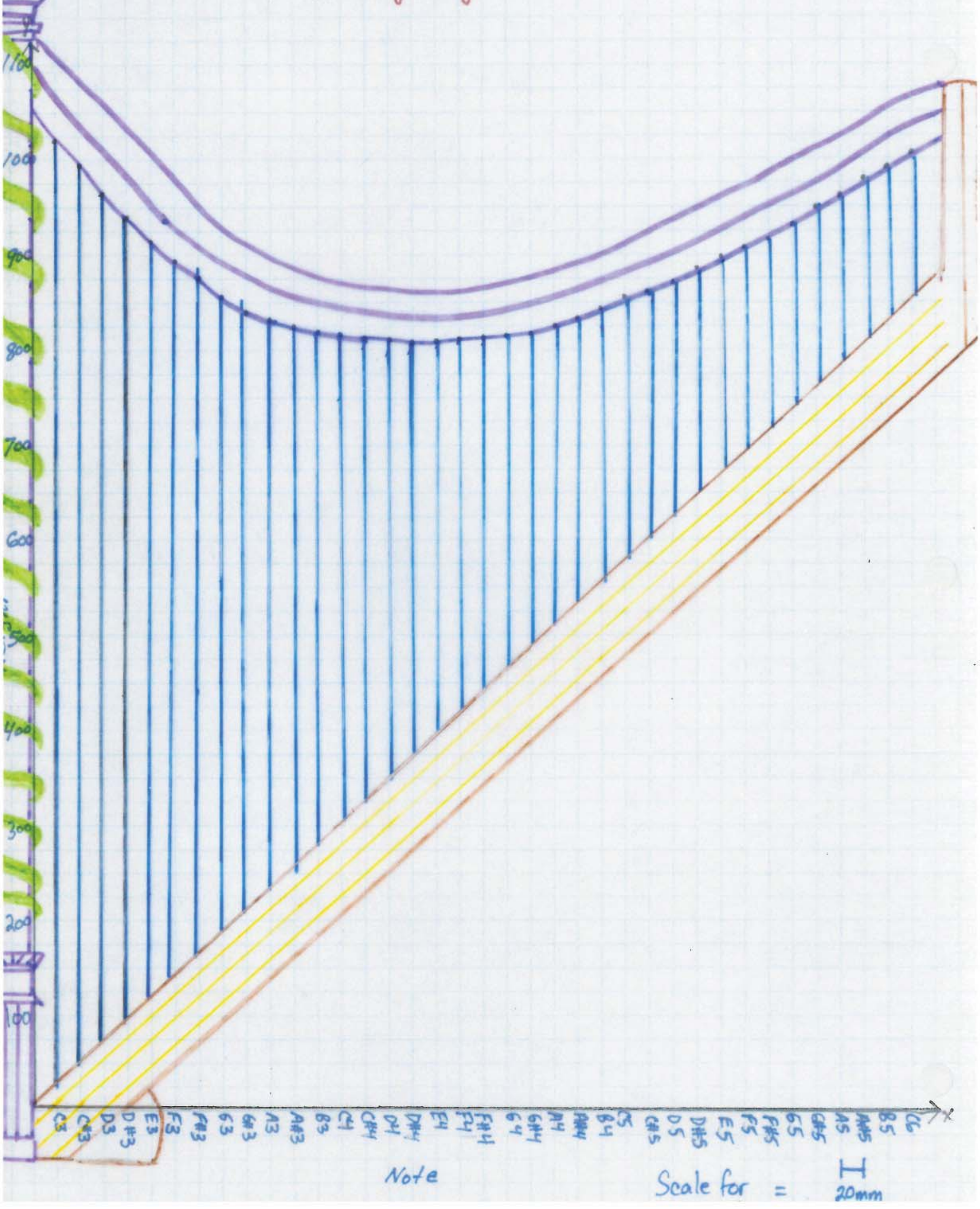
x= string # after the first string


Harp Lengths
1200
1000
800
600
400
200
0
0
5
10
15
20
25
30
35
40
Distance from the Base of Harp
Notes
Lengths



Harp Lengths
1200
1000
800
600
400
200
0
Distance from the Base of Harp
C3
D3
E
F#3
G#3
A#3
C4
D4
E4
F#4
G#4
A#4
C5
D5
E5
F#5
G#5
A#5
C6
Notes
Lengths

Harp String Lengths



Guitar Design and Mathematics

Fret Placement from the Bridge of the Guitar

Nut - *650mm*

Fret 1 - 650*.943874327= *613.5183126 mm*

Fret 2 - 613.5183126*.943874327 = *579.0841844 mm*

Fret 3 - 579.0841844*.943874327 = *546.5826948 mm*

Fret 4 - 546.5826948*.943874327 = *515.9053732 mm*

Fret 5 - 515.9053732*.943874327 = *486.9498369 mm*

Fret 6 - 486.9498369*.943874327 = *459.6194496 mm*

Fret 7 - 459.6194496*.943874327 = *433.8229987 mm*

Fret 8 - 433.8229987*.943874327 = *409.4743909 mm*

Fret 9 - 409.4743909*.943874327 = *386.4923651 mm*

Fret 10 - 386.4923651*.943874327 = *364.800221 mm*

Fret 11 - 364.800221*.943874327 = *344.3255631 mm*

Fret 12 - 344.3255631*.943874327 = *325.0000592 mm*

Fret 13 - 325.0000592*.943874327 = *306.7592121 mm*

Fret 14 - 306.7592121*.943874327 = *289.5421449 mm*

Fret 15 - 289.5421449*.943874327 = *273.2913971 mm*

Fret 16 - 273.2913971*.943874327 = *257.9527336 mm*

Fret 17 - 257.9527336*.943874327 = *243.4749628 mm*

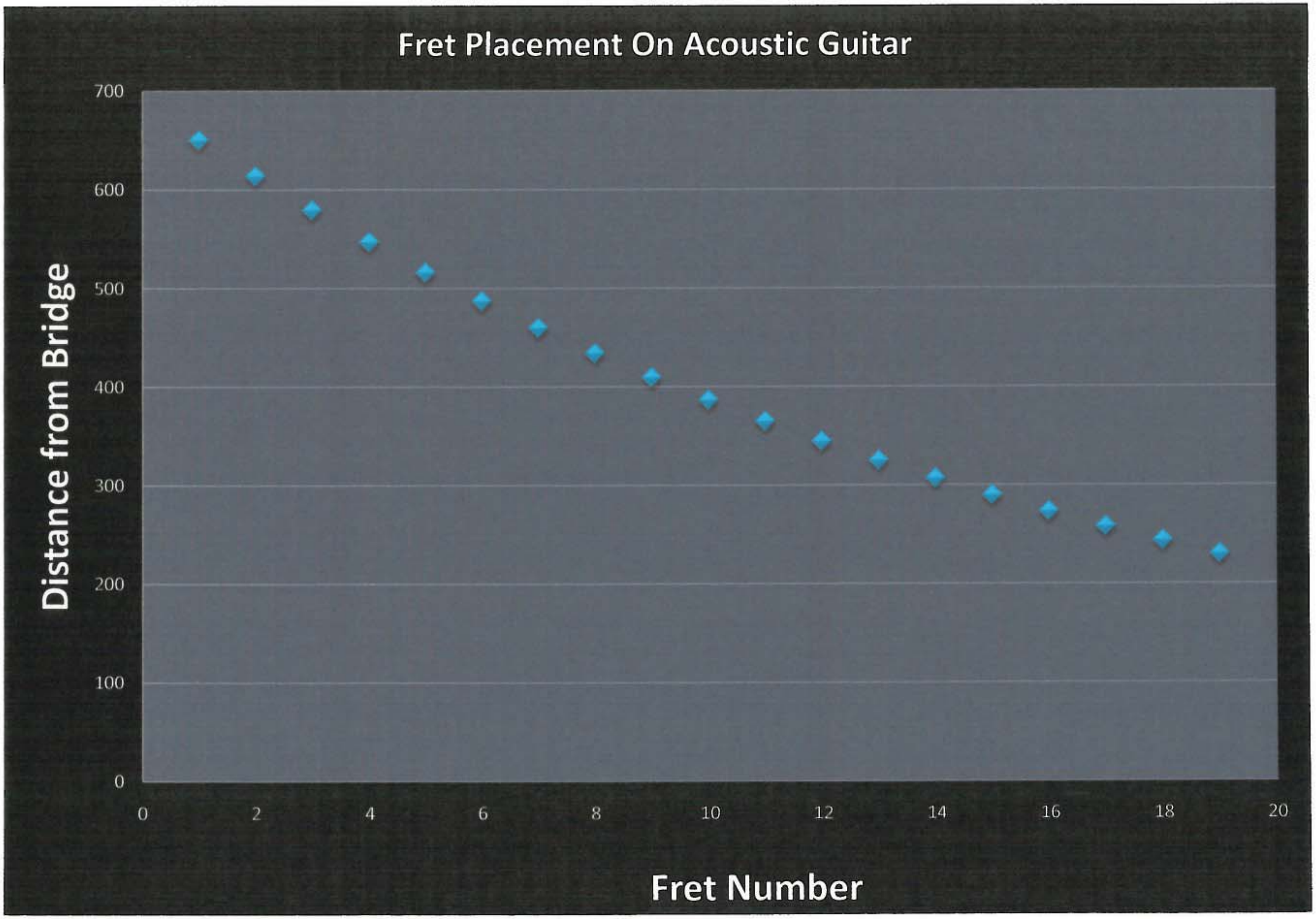
Fret 18 - 243.4749628*.943874327 = *229.8097666 mm*

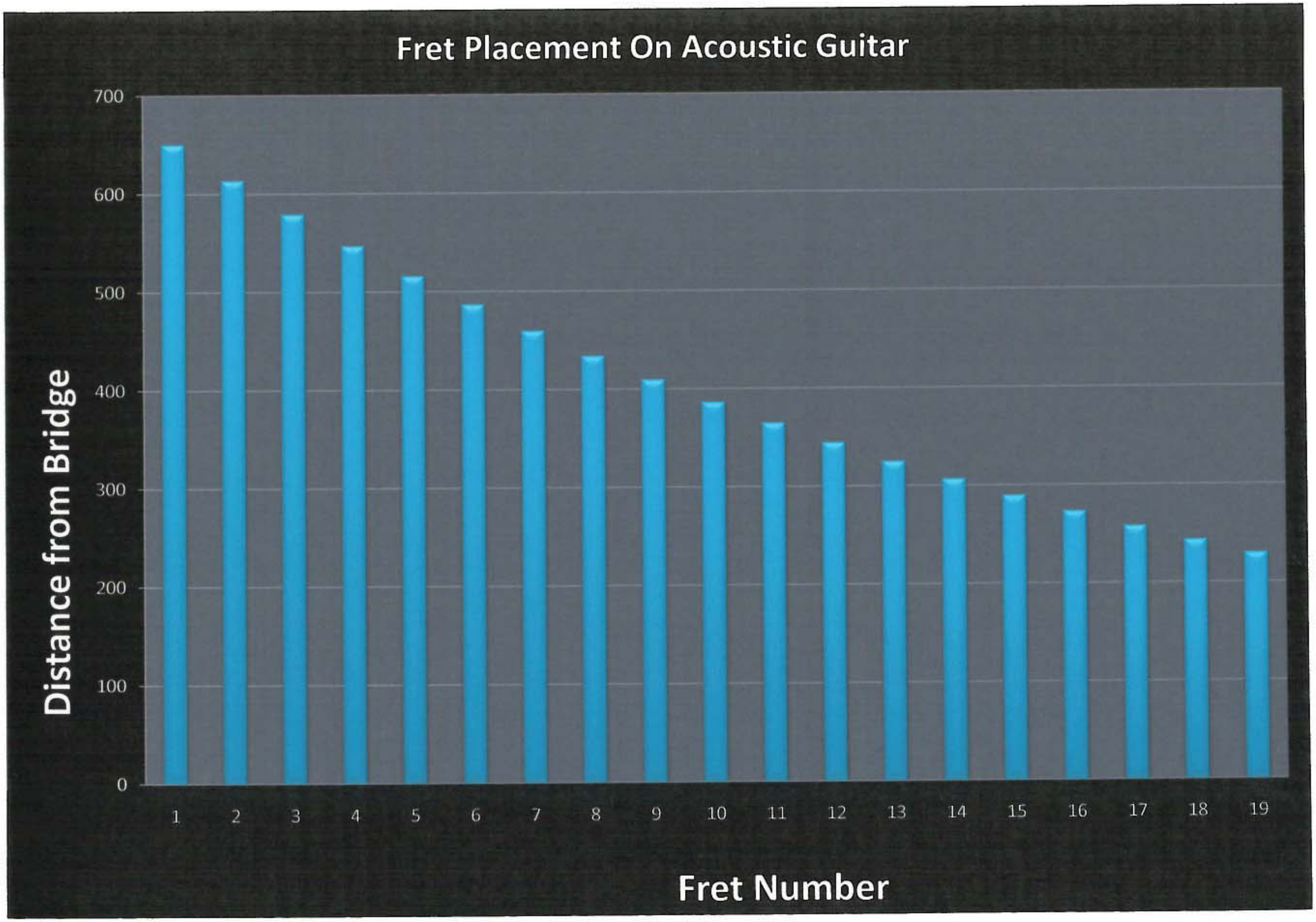
Rule of exponential decay used:

Y = 650(.943874327) ^X

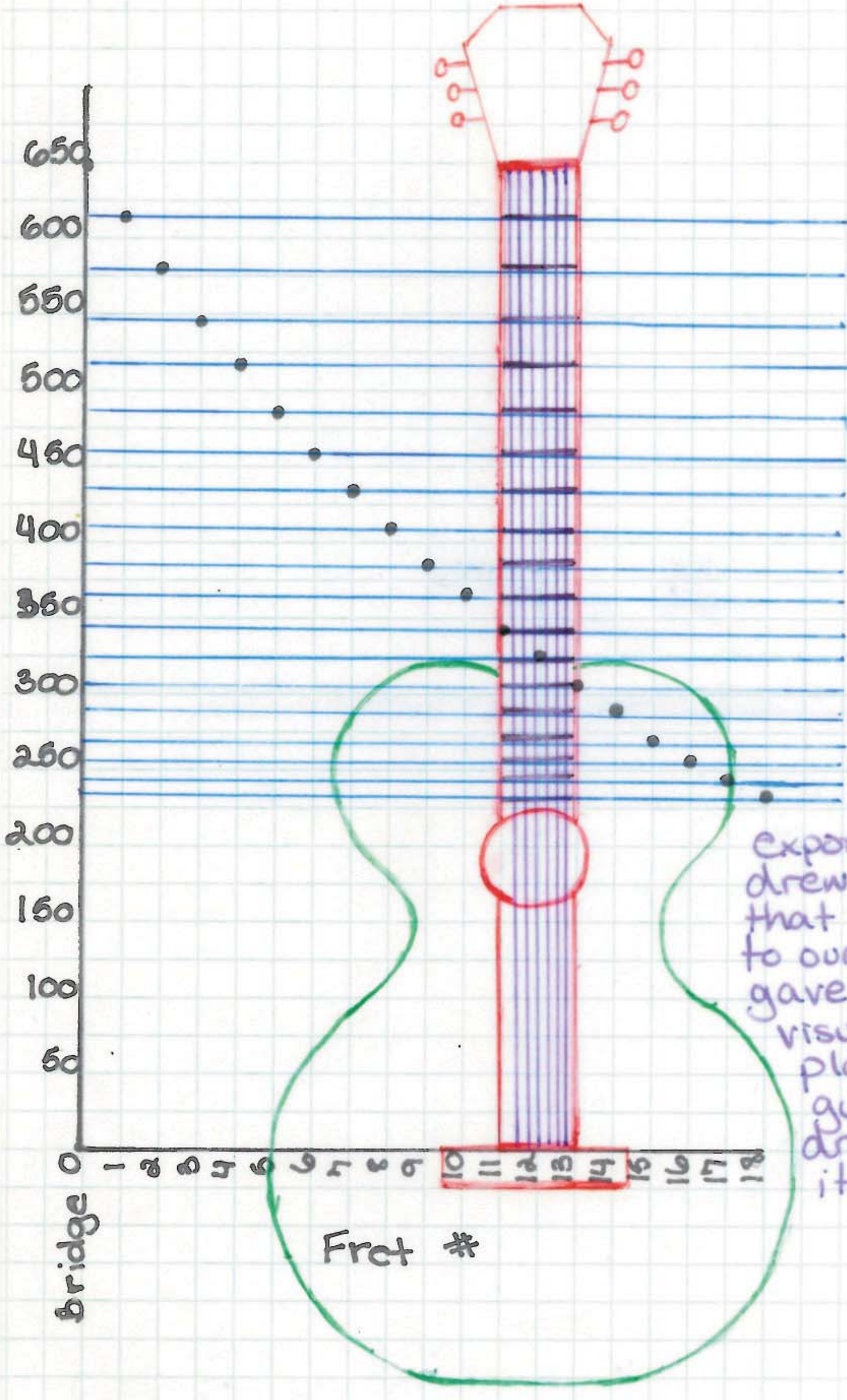
Note Frequencies for the Guitar

Notes	MIDI #	Frequencies
F2	41	6.875*2^((3+41)/12)=87.30705786
F#2	42	6.875*2^((3+42)/12)=92.49860568
G2	43	6.875*2^((3+43)/12)=97.998859
G#2	44	6.875*2^((3+44)/12)=103.8261744
A2	45	6.875*2^((3+45)/12)=110
A#2	46	6.875*2^((3+46)/12)=116.5409404
B2	47	6.875*2^((3+47)/12)=123.4708253
C3	48	6.875*2^((3+48)/12)=130.8127827
C#3	49	6.875*2^((3+49)/12)=138.5913155
D3	50	6.875*2^((3+50)/12)=146.832384
D#3	51	6.875*2^((3+51)/12)=155.5634919
E3	52	6.875*2^((3+52)/12)=164.8137785
F3	53	6.875*2^((3+53)/12)=174.6141157
F#3	54	6.875*2^((3+54)/12)=184.9972114
G3	55	6.875*2^((3+55)/12)=195.997718
G#3	56	6.875*2^((3+56)/12)=207.6523488
A3	57	6.875*2^((3+57)/12)=220
A#3	58	6.875*2^((3+58)/12)=233.0818808
B3	59	6.875*2^((3+59)/12)=246.9416506
C4	60	6.875*2^((3+60)/12)=261.6255653
C#4	61	6.875*2^((3+61)/12)=277.182631
D4	62	6.875*2^((3+62)/12)=293.6647679
D#4	63	6.875*2^((3+63)/12)=311.126987
E4	64	6.875*2^((3+64)/12)=329.6275569
F4	65	6.875*2^((3+65)/12)=349.2282314
F#4	66	6.875*2^((3+66)/12)=369.9944227
G4	67	6.875*2^((3+67)/12)=391.995436
G#4	68	6.875*2^((3+68)/12)=415.3046977
A4	69	6.875*2^((3+69)/12)=440
A#4	70	6.875*2^((3+70)/12)=466.1637615
B4	71	6.875*2^((3+71)/12)=493.8833013
C5	72	6.875*2^((3+72)/12)=523.2511306
C#5	73	6.875*2^((3+73)/12)=554.3645262
D5	74	6.875*2^((3+74)/12)=587.3295358
D#5	75	6.875*2^((3+75)/12)=622.2539674
E5	76	6.875*2^((3+76)/12)=659.2551138
F5	77	6.875*2^((3+77)/12)=698.4564659
F#5	78	6.875*2^((3+78)/12)=739.98888454
G5	79	6.875*2^((3+79)/12)=783.990872
G#5	80	6.875*2^((3+80)/12)=830.6093952
A5	81	6.875*2^((3+81)/12)=880


Fret Placement On Acoustic Guitar
Distance from Bridge
0
100
200
300
400
500
600
700
0
2
4
6
8
10
12
14
16
18
20
Fret Number


Fret Placement On Acoustic Guitar
Distance from Bridge
0
100
200
300
400
500
600
700
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
Fret Number

Fret Placement on Acoustic Guitar



This is a visual representation of how we were able to calculate the fret positions on our guitar. As the note frequencies increased exponetially, the fret positioning decreased exponentially. We drew straight lines that were parallel to our x. axis. This gave us a clear visual of the fret placement. The guitar was then drawn to show its appearance.

Guitar Measurements

Total Width

1. Radius=5.5 in.
2. Diamter=11 in.

5.5+3+5.5= **14 in**

Radius of Smaller Circle

$a^2 + b^2 = c^2$

$4^2+6.5^2= C^2$

16+42.25=58.25

$C^2=58.25$

$\sqrt{58.25}= \sqrt{c^2}$

C=7.632168761 7.632168761-5.5= **2.132168761 in.**

Radius of Medium Circle

$a^2 + b^2 = c^2$

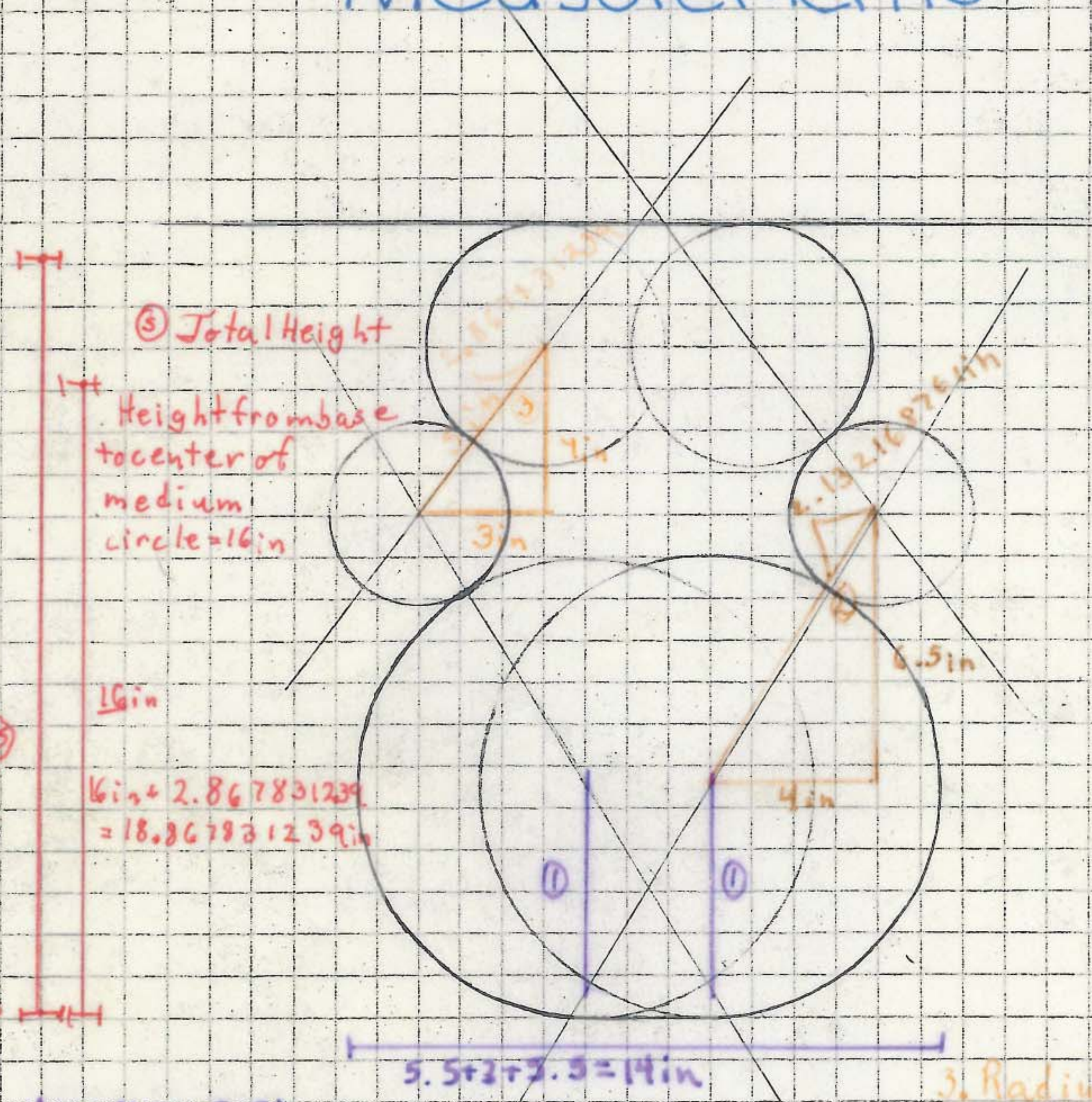
$3^2+4^2=c^2$

$25=c^2$

5=c — Distance from center of small circle to the center of the larger circle.

5-2.132168761= **2.867831139 in.**—**Radius of medium circle**

Guitar Measurements



5.5+3+5.5=14in

Total Width

1. radius=5.5in
diameter=11in

2. Radius of smaller circle

$a^2+b^2=c^2$

$4^2+6.5^2=c^2$

$16+42.25=58.25$

$c^2=58.25$

$\sqrt{58.25}=\sqrt{c^2}$

$c=7.632168761$

$7.632168761-5.5$

$=2.132168761in$

3. Radius of Medium Circle

$a^2+b^2=c^2$

$3^2+4^2=c^2$

$25=c^2$

$5=c$

Distance from center of small circle to center of larger circle

[illegible]

Sector Angles of the Guitar

Central Angle of medium circle:

The black dotted triangle is congruent to the purple dotted triangle. The dotted purple triangle is similar to the 3-4-5 triangle, so it has equivalent angles.

$180° - 36.86989765° = 143.1301301024°$

Central Angle for large circle:

$\text{Tan}^{-1}(6.5/4) = X$

$58.39249775 = x$

$58.39249775 + 90 = 148.3924978°$

Central Angle of smallest circle:

$\tan^{-1}(3/4) = X$

$36.86989765° = X$

$180 - (36.86989765 * 2)$

$106.2602047° = X$

Total Height

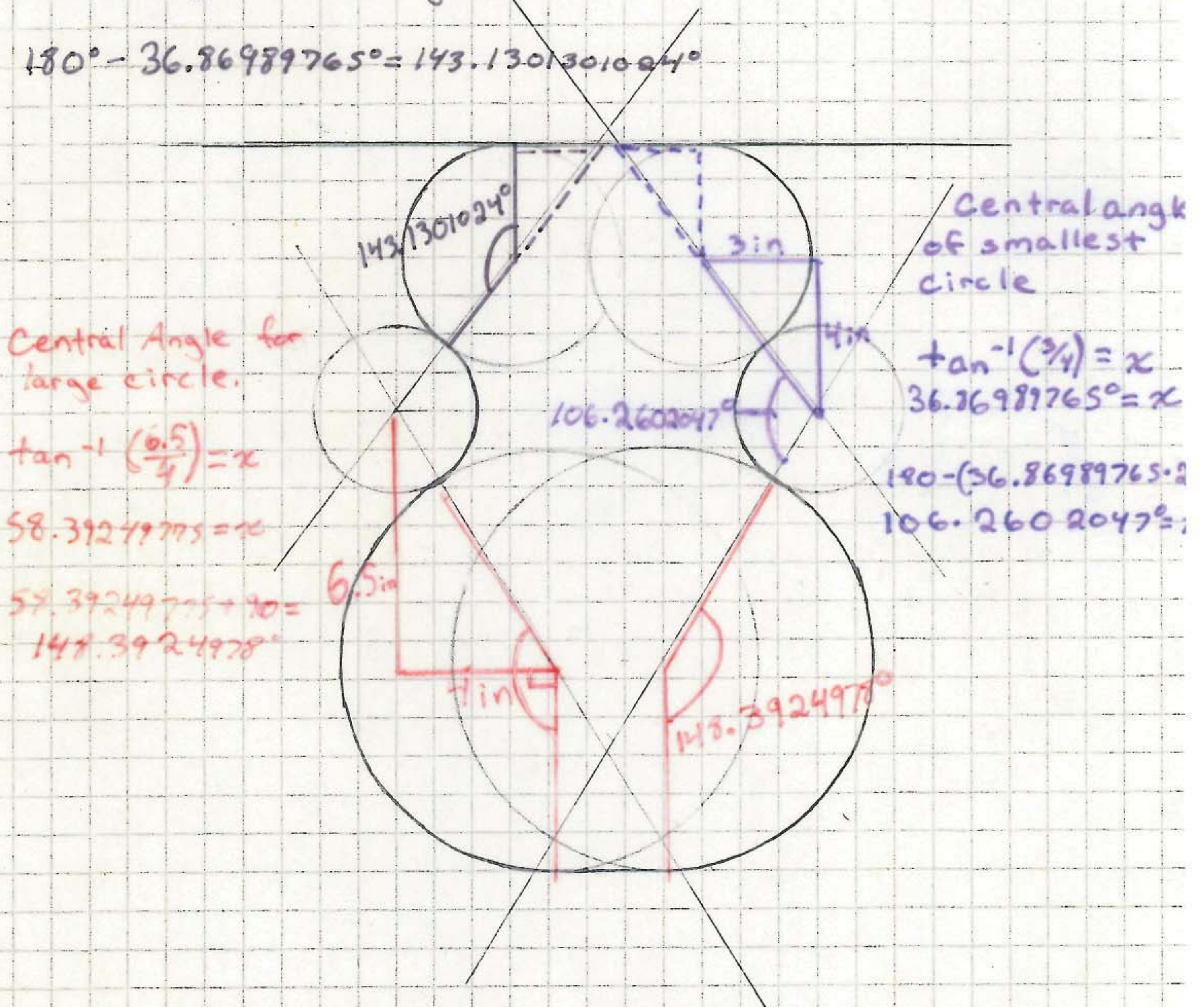
Height from base to center of medium circle= 16 in

16+2.867831239= **18.867831239 in.**

Central Angle of medium Circle

The dotted triangle is congruent to the dotted purple triangle. The dotted purple triangle is similar to the 3-4-5 triangle, so it has equivalent angles.

$180° - 36.86989765° = 143.1301010024°$



Sector Angles Of the Guitar

Perimeter of Guitar Sound Board

Circumference of large circle

2π (5.5)=34.55751919 in.

$$\frac{148.3924838^\circ}{34.55751919} = \frac{X}{360}$$

X=14.2446572in.= ARC LENGTH

Purple Section

Circumference of small circle

2π(2.132168761)*13.39681143 in.

$$\frac{106.2602047}{360} = \frac{X}{13.39681143}$$

X=3.954299792 = ARC LENGTH

Circumference of medium circle

$2\pi(2.867831239)=18.0191151$ in.

$$\frac{143.1301024}{360} = \frac{X}{18.0191151}$$

X=7164104972 in. = ARC LENGTH

Total Perimeter:

2(14.2446572)+2(3.954299792)

+2(7.164104972)+5+3=

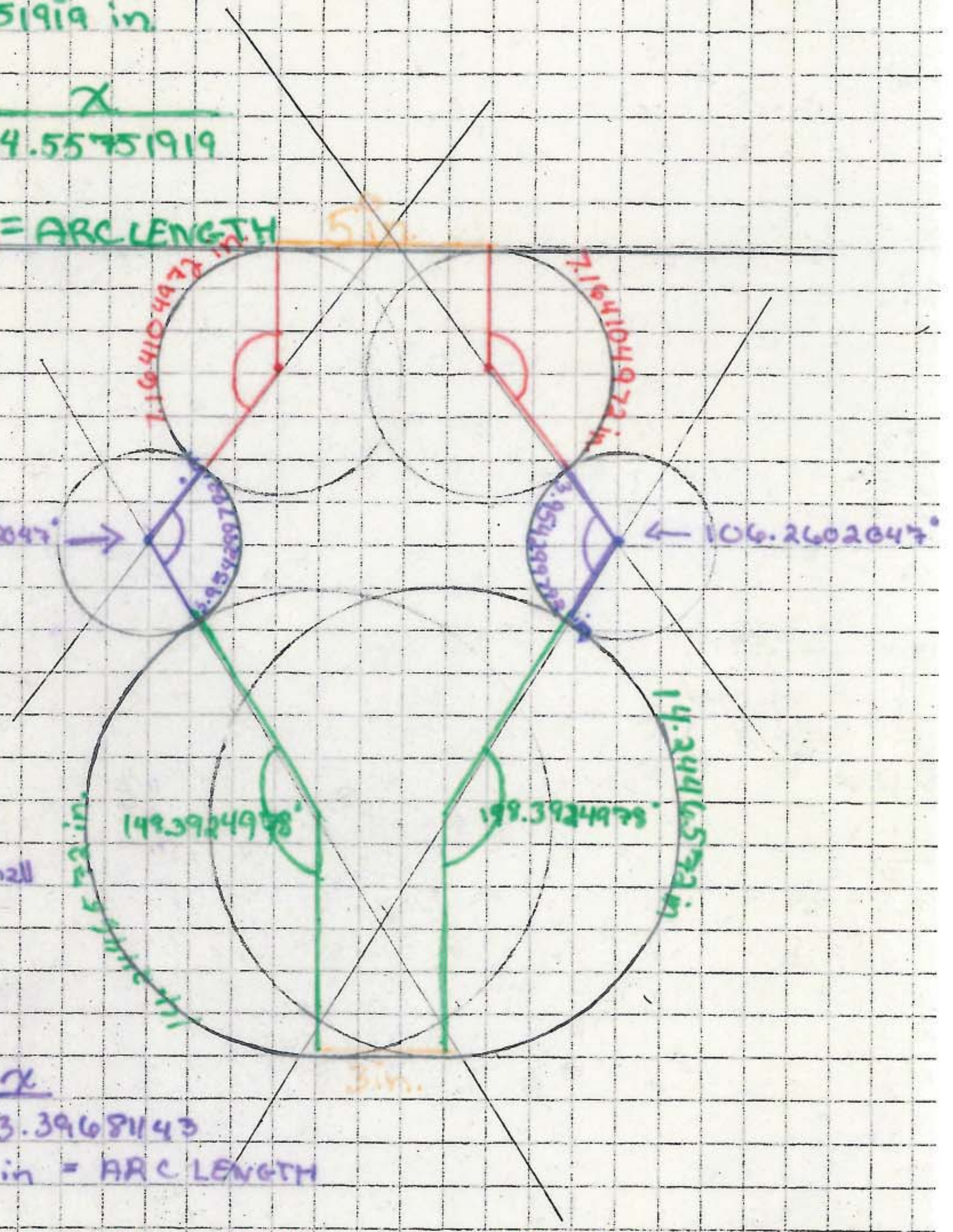
58.72612393 in.

Circumference of large circle

$2\pi(5.5) = 34.55751919$ in.

$$\frac{148.3924978^\circ}{360} = \frac{x}{34.55751919}$$

$x = 14.2446572$ in = ARC LENGTH



Purple Section

Circumference of small circle

$2\pi(2.132168761) =$

13.39681143 in

$$\frac{106.2602047}{360} = \frac{x}{13.39681143}$$

$x = 3.954299792$ in = ARC LENGTH

Circumference of medium circle

$2\pi(2.867831239) =$

18.0191151 in

$$\frac{143.1301024}{360} = \frac{x}{18.0191151}$$

$x = 7.164104972$ in = ARC LENGTH

Total Perimeter:

$2(14.2446572) + 2(3.954299792)$

$+ 2(7.164104972) + 5 + 3 =$

58.72612393 in

AREA OF THE FACE GUITAR SOUNDBOX

Green Section

Area of the Circle = $\prod r^2$
$\prod * 5.5^2 = 95.03317777 \text{ in}^2$

Sector Area

$$\frac{148.3924978^\circ}{360^\circ} = \frac{x}{95.03317777\text{in}^2}$$

$148.3924978 * 95.03317777 \div 360 = 39.17280727\text{in}^2$

Each Sector in Green is 39.17280727in^2 $39.17280727 \times 2 = 78.34561454 \text{ in}^2$
Total area is 78.34561454 in^2

Area of the Red Section

Area of Sectors
$106.2602047^\circ - 58.39249775^\circ = 47.86770695^\circ$

$$\frac{47.86770695^\circ}{360^\circ} = \frac{x}{14.28213142\text{in}^2}$$

$47.86770695 * 14.28213142 \div 360 = 1.899035783\text{in}^2$
$1.899035783 * 2 = 3.798071566 \text{ in}^2$

The area of both red sectors combined is 3.798071566 in^2

Area of the Trapezoid = .5(h)(base1+base2)

$.5(4)(11+5) = 32\text{in}^2$
32- 3.7980714566
<u>28.20192843 in^2 = Area of the red section of the guitar.</u>

Area of Purple section

Area of trapezoid ½*6.5(3+11) = 45.5 in^2
Tan^{-1} (6.5/4)=58.39249775°

Small Circle Area

$\pi (2.132168761)^2 = 14.28213142$

Sector Area

$$\frac{58.39249775}{360} = \frac{x}{14.28213142}$$

58.39249775*14.28213142÷ 360 = 2.316581463 in^2

2.316581463*2 = 4.633162833 in^2

45.5 in^2- 4.633162833 in^2 = **40.86683707 in^2= Area of purple section**

Area of Black Section

Area of black circle
$\pi(2.867831239)^2$= 25.837890 in^2

$$\frac{143.1301024}{360} = \frac{x}{25.8378906}$$

143.1301024x25.8378906/360=10.27272202 in^2

Area of each black section. 10.27272202x2=20.54544404 in^2
The total area of black section is 20.545444404 in^2

Area of Orange Section

3x5.5=16.5 in^2

The total area of the Orange Section is 16.5 in^2

Area of Brown Section

2.86783129x5=14.3391562 in^2

The total area of the brown section is 14.3391562 in^2

Face of The Guitar

2(39.17280727)+40.86683707+28.20192843+2(10.27272202)+14.3391562+16.5=

The total area of the guitar is **196.6381365 in^2**

Given Section

Area of Circle

$\pi r^2 = A$

$\pi 5.5^2 = 95.03317777 \text{ in}^2$

Sector Area

$\frac{148.3924978}{360^\circ} = \frac{x}{95.03317777\text{in}}$

$148.3924978 \times 95.03317777 \div 360 = 39.17280727$

$x = 39.17280727$ each

Area of green sectors

Area of Purple section

Area of trapezoid

$\frac{1}{2} \cdot 6.5(3+11) = 45.5 \text{in}^2$

$\tan^{-1}\left(\frac{6.5}{4}\right) = 58.39249775^\circ$

Circle Area

$\pi(2.132168761)^2 = 14.28213142$

Sector Area

$\frac{58.39249775}{360} = \frac{x}{14.28213142}$

$58.39249775 \times 14.28213142 \div 360 = 2.316581463 \text{in}^2$

$2.316581463 \times 2 = 4.633162833 \text{in}^2$

$45.5\text{in}^2 - 4.633162833\text{in}^2 = 40.86683707\text{in}^2$ = Area of Purple section

Area of Red trapezoid

Area of Sectors

$106.2602047^\circ - 58.39249775$

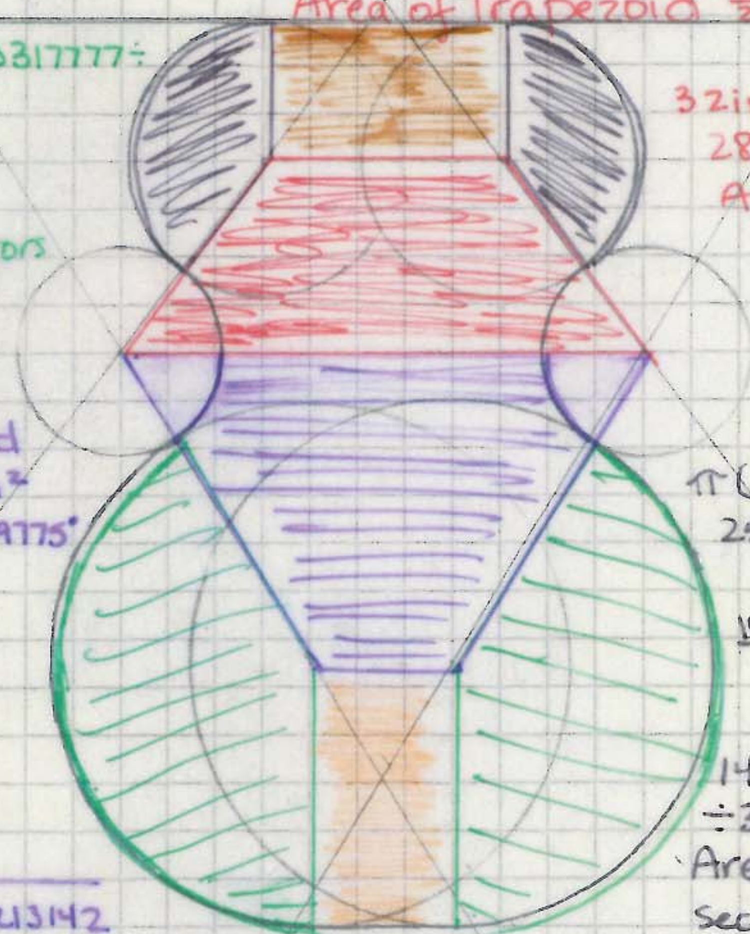
47.86770695°

$\frac{47.86770695}{360} = \frac{x}{14.28213142}$

$47.86770695 \times 14.28213142 \div 360 =$

$1.899035783 \times 2 = 3.798071566 \text{in}^2$

Area of Trapezoid $\frac{1}{2}(4)(11+5) = 32\text{in}^2$



$32\text{in}^2 - 3.798071566\text{in}^2$

28.20192843in^2

Area of Red section

Area of Black section

Area of Circle

$\pi(2.867831239)^2 \, 2$

25.8378906in^2

$\frac{143.1301024}{360} = \frac{x}{25.8378}$

$143.1301024 \times 25.83789 \div 360 = 10.27272202$

Area of each Black section

Area of Orange section

$3 \times 5.5 = 16.5 \text{in}^2$

Area of Brown section

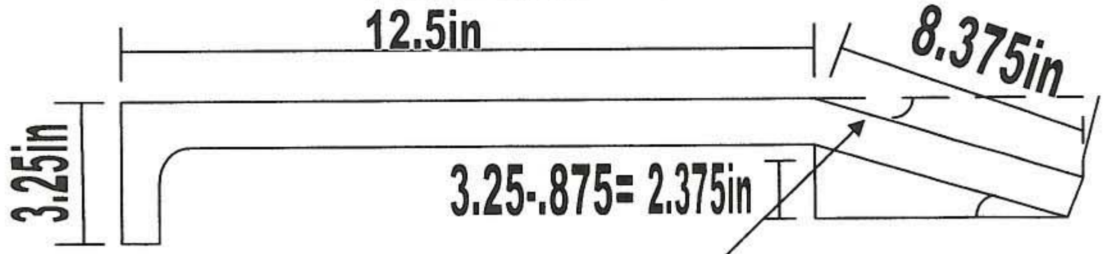
$2.867831239 \times 5 = 14.3391562 \text{in}^2$

Total Area

$2(39.17280727) + 40.86683707 + 28.20192843 + 2(10.27272202) + 14.3391562 + 16.5 =$

196.6381365in^2

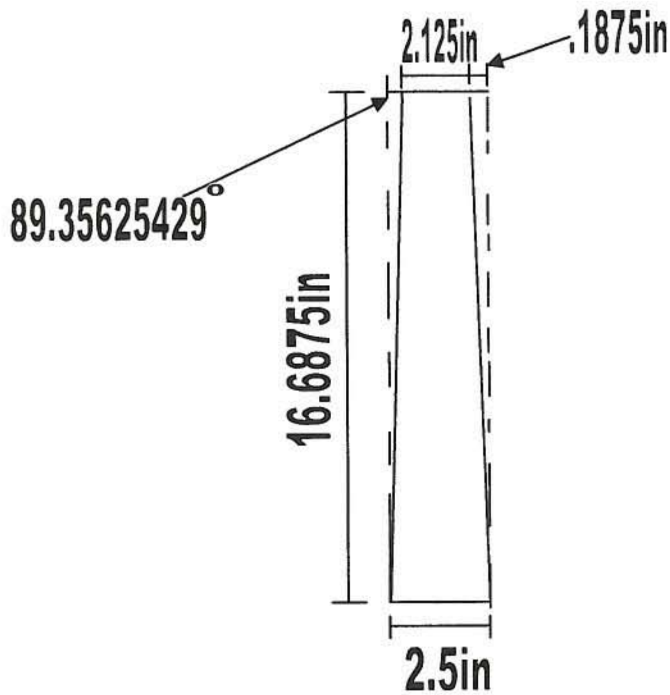
NECK MEASUREMENTS



$Sin^{-1}(2.375/8.375) = 16.47411189^{o}$

The angle of the handle is 16.47411189^{o}.

FINGER BOARD MEASUREMENTS

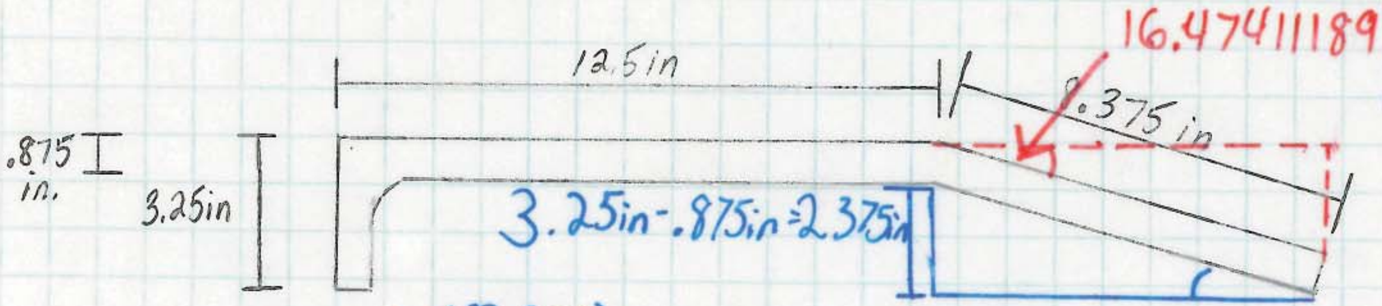


2.5-2.125=.375/2=.1875

$Tan^{-1}(16.6875/.1875) = 89.35625429$

The angle of the taper is 89.35625429^{o}

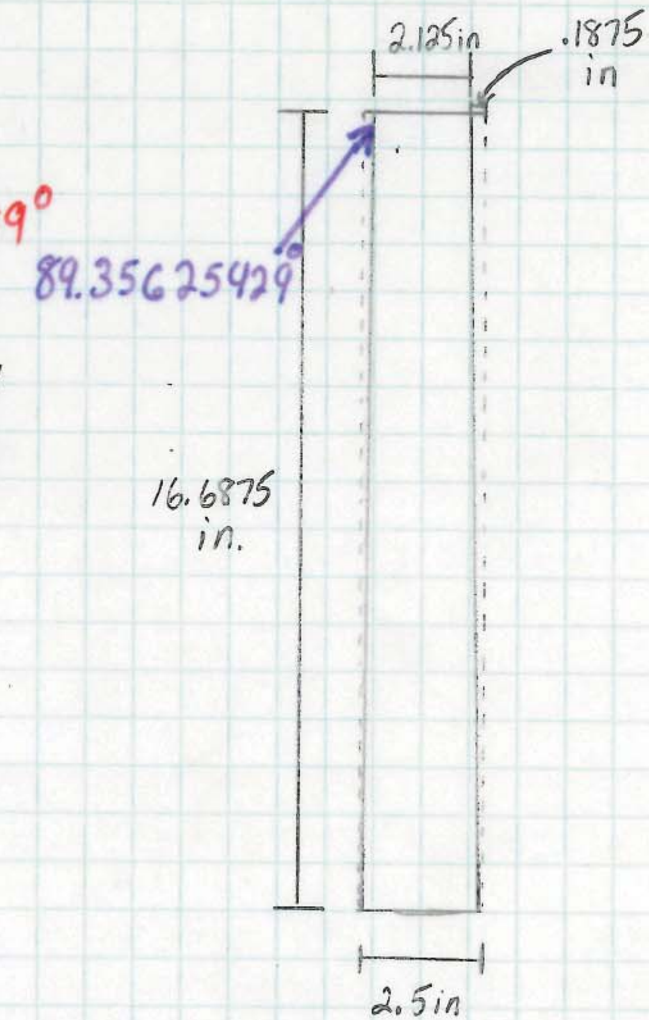
NECK MEASUREMENTS



$\sin^{-1}\left(\frac{2.375}{8.375}\right) = 16.47411189°$

The angle of the handle is 16.47411189°.

FINGER BOARD MEASUREMENTS



$2.5in - 2.125in = \frac{.375in}{2} = .1875$

$\tan^{-1}(16.6875/0.1875) = 89.35625429°$